A Guidance Law for General Surface Targets

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A modified pursuit guidance law is proposed for low-cost tactical missiles. Analysis of this law is carried out for surface targets with constant crossing velocity and constant crossing acceleration, and for stationary targets in the presence of gravity. The law is simple to implement using the velocity vane pursuit guidance seeker, and for the above targets is as accurate as proportional navigation. It is shown how classical pursuit guidance may seriously influence the stability of low-cost missiles which do not employ autostabilization. The present scheme significantly reduces this problem.

Introduction

THE proportional-navigation guidance law is used for most tactical missile applications involving high-speed maneuvering air targets.[1] However, against stationary surface targets it is usually better to employ pursuit gudiance. There are two reasons for this. First, implementation of pursuit guidance, using the velocity-vane or weathercock-stabilized seeker, is very simple.[2,3] Second, in the presence of gravity, pursuit guidance yields a stiffer or more rectilinear flight path to the target. During stretchout maneuvers, the likelihood that the missile will hit the ground before hitting the target is therefore reduced.[4]

Pursuit guidance is unsatisfactory against moving surface targets, and proportional navigation is generally used.[5,6] However, since this guidance law was developed for high-speed maneuvering air targets, it is not entirely satisfactory for slow-moving surface targets, mainly because the implementation of proportional navigation is relatively complex, particularly when gravity compensation is necessary.

Thus there is a need for a new guidance law for moving surface targets. This guidance law should be more accurate than pursuit guidance and easier to implement than proportional navigation. Previous modifications to pursuit or new schemes such as proportional lead guidance do not meet these requirements.[7] In contrast, the guidance law presented here, while it retains much of pursuit's simplicity, gives excellent performance against all surface targets. In terms of missile control deflection, the law may be expressed as

$$\delta = k_p(\epsilon + \mu\gamma) \tag{1}$$

where, as shown in Fig. 1, ϵ is the pursuit error, k_p and μ are constants, and γ is the flight heading. Implementation using modified Paveway pursuit guidance seekers is thus particularly simple.[8] As shown in Fig. 2, basic pursuit guidance is employed following target acquisition until the error angle ϵ is about 1 deg; the canards are then nulled and the missile weathercocks rapidly into coincidence with the velocity vector. At this time a two-degree-of-freedom reference gyro is uncaged, which defines the subsequent flight leading γ of the missile. Our main purpose in this report is to describe the theoretical performane of the modified pursuit guidance law assuming ideal, noise-free conditions.

Figure 3 shows a common implementation scheme for proportional-navigation guidance.[4] The precessional motion of the spin-stabilized optical assembly is $\dot{\theta}_g = T/H$, where H is spin momentum and T is gimbal motor torque. Making T proportional to ϵ yields $\dot{\theta}_g = k_g\epsilon = k_g(\theta_t - \theta_g)$. Hence,

$$\dot{\theta}_g = \frac{\dot{\theta}_t}{1 + p/k_g} \tag{2}$$

where p denotes d/dt. Thus, except for the time lag, $k_g\epsilon$ provides a direct measure of sight-line turning rate. Employing

$$k_p = \frac{\delta}{\epsilon} \quad \text{and} \quad k_a = \frac{\dot{\gamma}}{\delta} \tag{3}$$

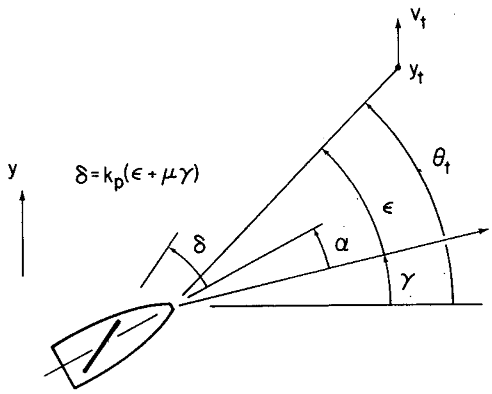

Fig. 1 The guidance law.

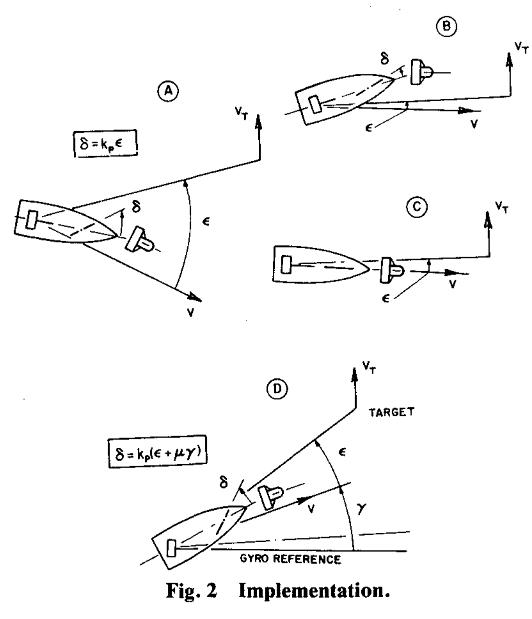

Fig. 2 Implementation.

Received Aug. 13, 1981; revision received Jan. 17, 1983.

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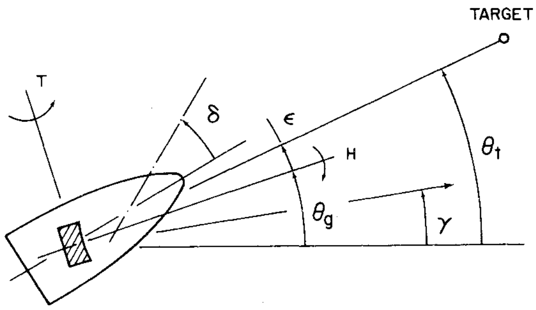

Fig. 3 Proportional navigation.

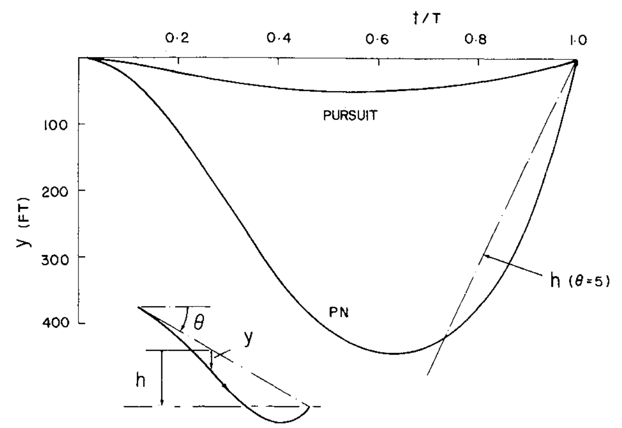

Fig. 4 Trajectory curvature due to gravity.

there follows

$$\dot{\gamma} = \frac{\beta}{k_g}\dot{\theta}_t \tag{4}$$

where

$$\beta = k_p k_a \tag{5}$$

This defines the proportional-navigation constant as $N = \beta / k_g$.

The control law for pursuit guidance follows from Eq. (1) with $\mu = 0$. This guidance law is easier to implement than proportional navigation, particularly when the simple velocity-vane or weathercock-stabilized seeker is used to establish ϵ.

The effect of gravity on flight-path curvature for proportional navigation ($\dot{\gamma} = 4\dot{\theta}_t$) and pursuit ($\dot{\gamma} = 4\epsilon$) is shown in Fig. 4. Comparison of y and h for the proportional-navigation missile indicates contact with the ground several thousand feet short of the target.‡ Indeed, with T now denoting total time of flight, this will always occur when[4]

$$\frac{Tg}{(N-1)\,V\sin\theta} > 1 \tag{6}$$

Accordingly, the proportional-navigational missile is most likely to rquire gravity compensation in roles involving extended shallow dive attacks at subsonic or low supersonic speeds. Of course, the device of increasing N to reduce y is not acceptable.

‡These calculations assume a $4g$ lateral acceleration limit, well above the maximum proportional navigation g command $N\cos\theta/N-2$. With pursuit guidance, the g command is equal to the inverse of time to go.

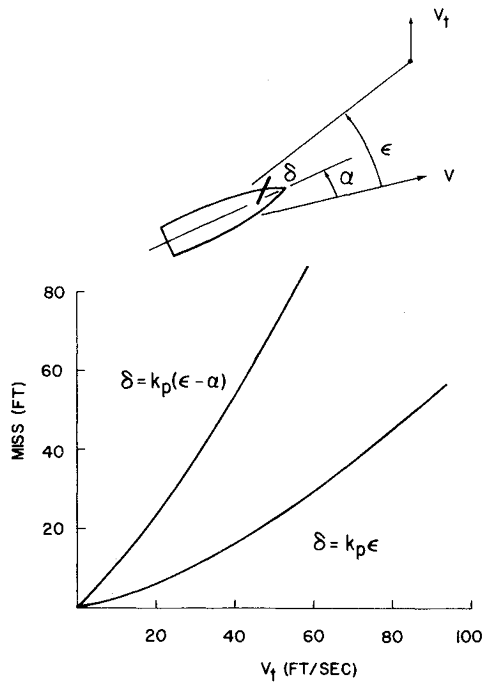

Fig. 5 Miss for pursuit guidance.

In contrast, the pursuit trajectory, also shown in Fig. 4, exhibits much less gravity sag. In this case, contact with the ground occurred just 0.05 s before anticipated target impact. Such results emphasize that in the absence of gravity compensation pursuit guidance may outperform proportional navigation.

But pursuit guidance is entirely unsatisfactory with target motion present. For example, the miss distance against targets with constant crossing velocity is

$$\text{miss} = (1 + r\beta)\frac{V_t}{\beta} + \frac{V_t^2}{2\bar{a}} \tag{7}$$

where $\bar{a}$ is the lateral acceleration limit and r is either zero (velocity pursuit) or unity (body-fixed pursuit). With $\bar{a} = 4g$, Eq. (7) is shown in Fig. 5§

Theoretical Development

This analysis treats targets with constant crossing velocity, constant crossing acceleration, and stationary targets in the presence of gravity. The crossing cases are considered because they generally demand maximum lateral acceleration capability. The analysis also assumes small deviations from constant-bearing geometry, enabling small-angle approximations to be used throughout. Then, from Eq. (1) and Fig. 1,

$$\delta = k_p[\theta_t + (\mu - 1)\gamma] \tag{8}$$

where $\theta_t = (y_t - y)/(T-t)V$, T denotes initial time to go, and $\gamma = \dot{y}/V$. Also, from Eq. (3), with a correction for gravity, $\dot{\gamma} = k_a\delta - g/V$. Thus,

$$\ddot{y} + (1-\mu)\beta\dot{y} + \frac{\beta y}{T-t} = \frac{\beta y_t}{T-t} - g \tag{9}$$

is the differential equation describing the guidance law. For surface targets engaged close in, it is sufficient to assume

$$y_t = V_t t + a_t\frac{t^2}{2} \tag{10}$$

§The calculations in this report generally assume $T = 10$ or 15 s, $\beta = 4/\text{s}$, and $\theta = 5$ deg.

Also, it is more convenient to use the miss coordinate

$$m = y - y_t(T) \tag{11}$$

which from Eqs. (9) and (10) satisfies

$$\ddot{m} + (1-\mu)\beta\dot{m} + \frac{\beta m}{T-t} = -\beta V_t + \frac{a_t}{2}(T-t) - g \tag{12}$$

Introducing the new independent variable

$$\zeta = (1-\mu)\beta(T-t) \tag{13}$$

Equation (12) becomes

$$\ddot{m} - \dot{m} + \frac{1}{1-\mu}\frac{m}{\zeta} = \frac{-\beta(V_t + a_t T) - g}{(1-\mu)^2\beta^2} + \frac{a_T\zeta}{2(1-\mu)^3\beta^2} \tag{14}$$

This has the solution

$$m(\zeta) = e^{\zeta/2}[aW_{\lambda\eta}(\zeta) + bW_{-\lambda\eta}(-\zeta)] - \frac{1-\mu}{\mu}\left[\frac{\beta(V_t + a_t T) + g}{(1-\mu)^2\beta^2} + \frac{1-\mu}{2-\mu}\frac{a_t}{(1-\mu)^3\beta^2}\right]\zeta + \frac{a_t\zeta^2}{2(2\mu-1)(1-\mu)^2\beta^2} \tag{15}$$

where $W_{\lambda\eta}$ is the Whittaker confluent hypergeometric function,[9] with $\eta = 1/2$ and $\lambda = (1-\mu)^{-1}$.

In keeping with the near-constant bearing trajectory, assume zero initial conditions. Then, using asymptotic properties of the Whittaker function for large ζ_0, the miss is

$$m(0) = -a\Gamma(\lambda)\sin(\pi\lambda) \tag{16}$$

where $\zeta_0 = (1-\mu)\beta T$ and a is a linear function of V_t, a_t, and g.

For a constant crossing velocity target

$$a = \frac{V_t\zeta_0^{\mu/\mu-1}}{\mu(1-\mu)\beta\pi} \tag{17}$$

and the miss, given by Eqs. (16) and (17), is shown in Fig. 6a. It is vanishingly small at the first four zeroes of $\sin(\pi\lambda)$; that is, for $\mu = 1/2$, $2/3$, $3/4$, and $4/5$. For $\mu = 0$ it reduces to V_t/β, the miss of the pursuit missile with unlimited lateral-acceleration capability. The dashed curve in Fig. 6a, obtained by integrating Eq. (14) numerically, shows the miss for the more practical case of a $4g$ acceleration limit. Lateral acceleration obtained by differentiating Eq. (15) is shown in Fig. 6b.

For a target with constant crossing acceleration

$$a = \frac{a_t(1-3/2\mu)\zeta_0^{2\mu-1/\mu-1}}{\mu(2\mu-1)(1-\mu)^2\beta^2\pi} \tag{18}$$

and the miss for this case, shown in Fig. 7, exhibits zeroes at $\mu = 2/3$, $3/4$, and $4/5$. These are the same as before except that $\mu = 1/2$ must now be excluded due to the presence of $(2\mu - 1)$ in the denominator of Eq. (18).

For the stationary target in the presence of gravity

$$a = \frac{g\zeta_0^{\mu/\mu-1}}{\mu(1-\mu)\beta^2\tan\theta\pi} \tag{19}$$

and the miss, which in this case is referred to the ground plane, is shown in Fig. 8. It is important to emphasize here

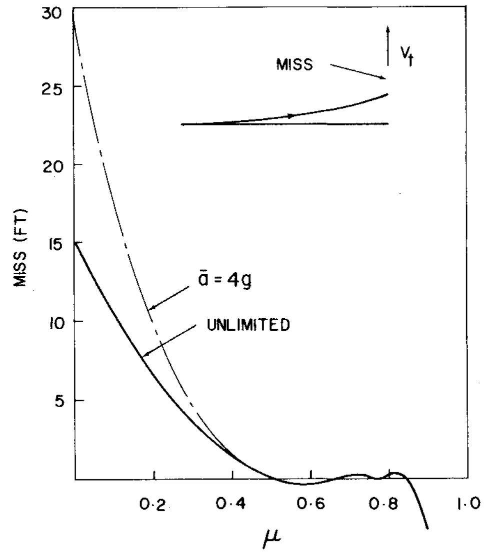

Fig. 6a Miss for $V_t = 60$ ft/s.

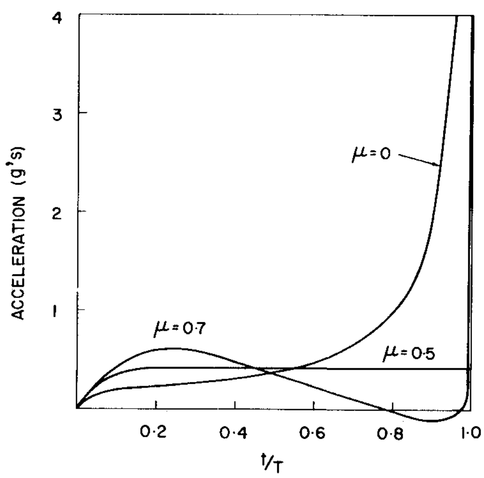

Fig. 6b Miss acceleration.

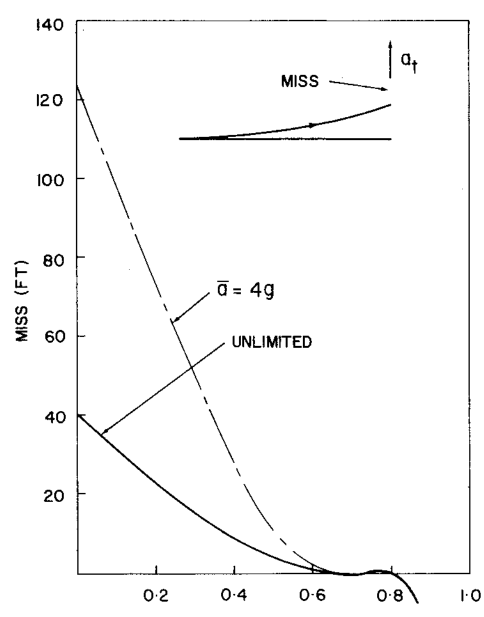

Fig. 7 Miss for $a_t = 16$ ft/s^2.

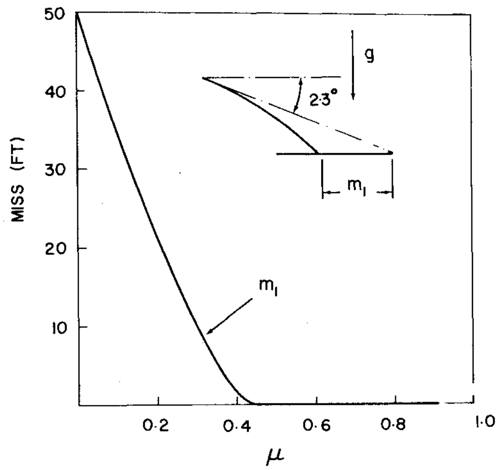

Fig. 8 Miss due to gravity.

that the general shape of the basic pursuit trajectory shown in Fig. 3 is little changed for μ about 1/2. Approximately zero miss is therefore possible with the new guidance law without gravity compensation.

Dynamic Stability

In the absence of autopilot stability, the pursuit-guided missile may experience sustained angle-of-attack oscillations. These oscillations, because of control deflection limiting, are generally of bounded magnitude. They have been observed repeatedly during guided-projectile flight tests at Dahlgren.[2] The influence of the present guidance law on stability is described below.

The oscillations of the missile with canard deflection δ are

$$I\ddot{\theta}_m - M_q\dot{\theta}_m - M_\alpha\alpha = M_\delta\delta \tag{20}$$

with the usual aerodynamic notation and where

$$\theta_m = \alpha + \gamma \tag{21}$$

as shown in Fig. 1. From Eqs. (20) and (21)

$$\ddot{\alpha} - 2\gamma_0\dot{\alpha} + \omega_0^2\alpha = \omega_0^2 r\delta \tag{22}$$

where the frequency, damping, and trim concepts are, respectively,

$$\omega_0^2 = -\frac{M_\alpha}{I} \tag{23}$$

$$\gamma_0 = \frac{1}{2}\left[\frac{M_q}{I} - \frac{F_\alpha}{mV}\right] \tag{24}$$

$$r = (\alpha/\delta)_{\text{trim}} \tag{25}$$

For oscillating angle of attack, the second part of Eq. (3) must be modified to read $\dot{\gamma} = k_a\alpha/r$. This enables Eq. (8) to be written

$$\dot{\delta} = k_p\left[\dot{\theta}_t + \frac{k_a}{r}(\mu - 1)\alpha\right] \tag{26}$$

For slow targets at large range, $\dot{\theta}_t$ in the above equation may be neglected compared with the angle-of-attack term. Equation (22), describing the angle-of-attack oscillations, thus becomes

$$\dddot{\alpha} - 2\gamma_0\ddot{\alpha} + \omega_0^2\dot{\alpha} + \omega_0^2\beta(1-\mu)\alpha = 0 \tag{27}$$

Now assume the following motion about trim

$$\alpha = e^{(i\omega_0 + \lambda)t} \tag{28}$$

where $\omega_0 \gg \lambda$. There follows

$$\lambda = \lambda_0 + \frac{\beta}{2}(1-\mu) \tag{29}$$

Thus, since $\lambda < 0$ for dynamic stability, implementation of velocity pursuit guidance ($\mu = 0$) is strongly destabilizing. Indeed, during guided projectile development at Dahlgren, β's of about 4 caused $\lambda = 0$. Clearly, stability must be considered carefully before attempting to reduce the pursuit miss by increasing β. Due to the term $(1-\mu)$ in Eq. (29), implementation of the new guidance law has a reduced effect on stability.

Conclusion

A guidance law has been presented which is particularly suited for ship-to-ship missile applications. For these targets the law possesses the best features of both pursuit and proportional-navigation guidance. With pursuit it results in small trajectory curvature in the presence of gravity. It is also easy to implement. With gravity-compensated proportional navigation, it results in near-zero miss.

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